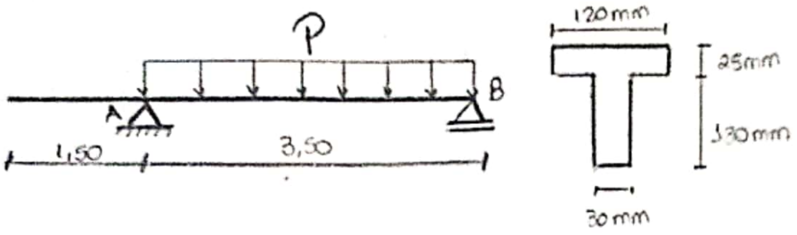
QUESTÃO 1. TIPO A.



$\sigma_{máx} = 28MPa$

$\tau_{máx} = 1,80 MPa$

- Reações de apoio:

$\circlearrowleft \Sigma M_B = 0 \rightarrow P.\ 3,5\ 1,75 - V_A \cdot 3,50 = 0 \rightarrow$ $\boxed{V_A = V_B = 1,75P}$

- cálculo do centróide da seção:

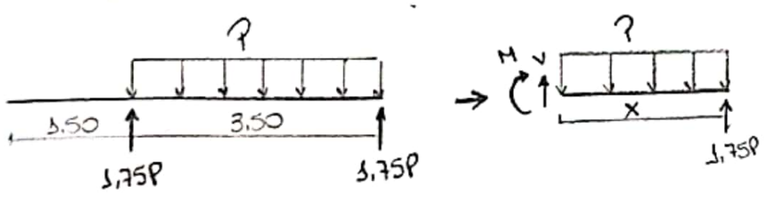
$$\bar{Y} = \frac{65 \cdot 30 \cdot 130 + 142,5 \cdot 120 \cdot 25}{30 \cdot 130 + 120 \cdot 25} \rightarrow \boxed{\bar{Y} = 98,696 = 98,70\ mm}$$

- Cálculo do momento de inércia

$$I = \left[\frac{0,03 \cdot 0,13^3}{12} + 0,03 \cdot 0,13 \cdot 0,0337^2\right] + \left[\frac{0,12 \cdot 0,025^3}{12} + 0,12 \cdot 0,025 \cdot 0,0438^2\right]$$

$$\boxed{I = 1,583 \cdot 10^{-5}\ m^4}$$

- Diagramas de momento fletor e esforço cortante.



$\circlearrowleft \Sigma M = 0 \rightarrow -M - \frac{P.x^2}{2} + 1,75P.x = 0$

$M = 1,75P.x - \frac{P.x^2}{2}$

$M_{máx}$ em $x = 1,75m$

$\boxed{M_{máx} = 1,531P}$

$V - Px + 1,75P = 0$

$V = Px - 1,75P \rightarrow V_{max} = x = 0$ ou $x = 3,5m \rightarrow \boxed{V_{máx} = 1,75P}$

- cálculo do maior valor de P

$$S_{req} = \frac{M_{máx}}{\sigma_{adm}} \rightarrow \frac{I}{\bar{Y}} = \frac{M_{máx}}{\sigma_{adm}} \rightarrow \frac{1,583 \cdot 10^{-5}}{0,0987} = \frac{1,531\ P}{28 \cdot 10^6} \rightarrow \boxed{P = 2,93\ KN/m}$$

- Verificação da Tensão de cisalhamento.

$$Q = \frac{0,0987}{2} \cdot 0,0987 \cdot 0,03 \rightarrow \boxed{Q = 1,46 \cdot 10^{-4}\ m^3}$$

$$\tau_{máx} = \frac{V_{máx} \cdot Q}{I \cdot t} = \frac{1,75 \cdot 2,93 \cdot 1,46 \cdot 10^{-4}}{1,583 \cdot 10^{-5} \cdot 0,03} \rightarrow \tau_{máx} = 1,58\ MPa < \tau_{adm} \rightarrow OK!$$

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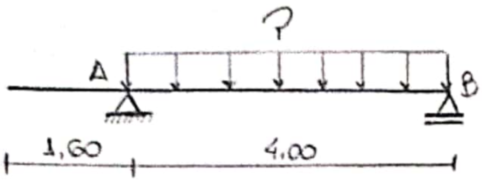
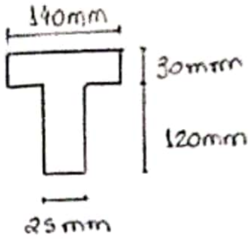
QUESTÃO 1. TIPO B





$\sigma_{adm} = 31\,MPa$

$\tau_{máx} = 1,80\,MPa$

- Reações de apoio:

$\circlearrowleft \sum M_B = 0 \rightarrow P \cdot 4,0 \cdot 2,0 - V_A \cdot 4,0 = 0 \rightarrow \boxed{V_A = V_B = 2,0P}$

- Cálculo do centróide da seção

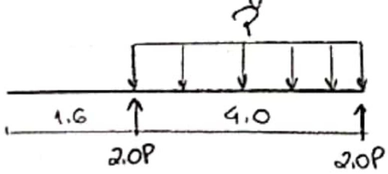
$$\bar{Y} = \frac{60 \cdot 120 \cdot 25 + 135 \cdot 140 \cdot 30}{25 \cdot 120 + 140 \cdot 30} \rightarrow \boxed{\bar{Y} = 103,75\,mm}$$

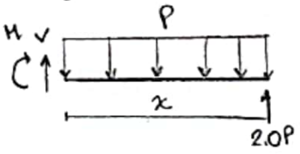
- Cálculo do momento de inércia

$$I = \left[\frac{0,025 \cdot 0,12^3}{12} + 0,025 \cdot 0,12 \cdot 0,04375^2\right] + \left[\frac{0,14 \cdot 0,03^3}{12} + 0,14 \cdot 0,03 \cdot 0,03125^2\right]$$

$$\boxed{I = 1,376 \cdot 10^{-5}\,m^4}$$

- Momento fletor e esforço cortante máximo.





$V - Px + 2,0P = 0$

$V = Px - 2,0P$

$V_{máx}$ em $x = 2,0$ ou $x = 0$ $\rightarrow \boxed{V_{máx} = 2,0P}$

$\sum M = 0 \rightarrow -M - \frac{Px^2}{2} + 2,0P\,x = 0$

$M = 2,0Px - \frac{Px^2}{2} \rightarrow M_{máx}$ em $x = 2,0 \rightarrow \boxed{M_{máx} = 2,0P}$

- Cálculo do maior valor de P:

$$S_{req} = \frac{M_{máx}}{\sigma_{adm}} \rightarrow \frac{I}{\bar{Y}} = \frac{M_{máx}}{\sigma_{adm}} \rightarrow \frac{1,376 \cdot 10^{-5}}{0,10375} = \frac{2,0P}{31 \cdot 10^6} \rightarrow \boxed{P = 2,06\,kN/m}$$

- Verificação da tensão de cisalhamento:

$$Q = \frac{0,10375}{2} \cdot 0,10375 \cdot 0,025 \rightarrow \boxed{Q = 1,346 \cdot 10^{-4}\,m^3}$$

$$\tau = \frac{V_{máx} \cdot Q}{I \cdot t} = \frac{2,0 \cdot 2056 \cdot 1,346 \cdot 10^{-4}}{1,376 \cdot 10^{-5} \cdot 0,025} \rightarrow \tau_{máx} = 1,61\,MPa < \tau_{adm} \rightarrow OK!$$

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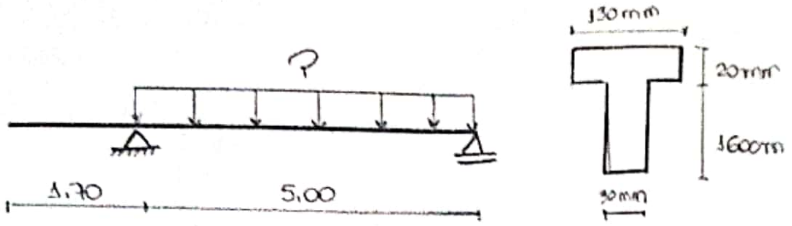
QUESTÃO 1. TIPO C



$\sigma_{adm} = 26\,MPa$

$\tau_{adm} = 1{,}80\,MPa.$

- Reações de apoio:

$\curvearrowleft + \Sigma M_B = 0 \rightarrow P.5{,}0\ 2{,}5 - V_A\ 5{,}0 \rightarrow \boxed{V_A = V_B = 2{,}5\,P}$

- Cálculo do centróide da seção:

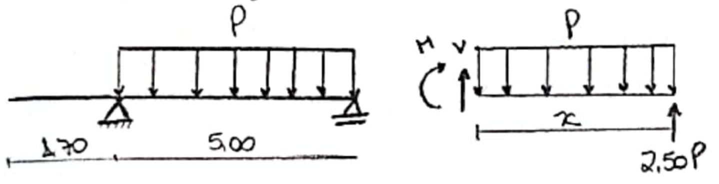
$$\bar{Y} = \frac{80.160.30 + 170.20.130}{160.30 + 130.20} \rightarrow \boxed{\bar{Y} = 111{,}62\ mm}$$

- Cálculo do momento fletor.

$$I = \left[\frac{0{,}03.\ 0{,}16^3}{12} + (0{,}03.\ 0{,}16.\ 0{,}03162^2)\right] + \left[\frac{0{,}13.\ 0{,}02^3}{12} + (0{,}13.\ 0{,}02.\ 0{,}05838^2)\right]$$

$$\boxed{I = 2{,}40.10^{-5}\ m^4}$$

- Momento fletor e esforço cortante máximo:



$V - Px + 2{,}5P = 0$

$V = Px - 2{,}5P \rightarrow \boxed{V_{máx} = 2{,}50\,P}$

$$\Sigma M = 0 \rightarrow -M - P\frac{x^2}{2} + 2{,}5Px = 0 \rightarrow M = 2{,}5Px - \frac{Px^2}{2} \rightarrow \boxed{M_{máx} = 3{,}125\,P}$$

- cálculo do maior valor de P:

$$S_{req} = \frac{M_{máx}}{\sigma_{adm}} = \frac{I}{\bar{Y}} = \frac{M_{máx}}{\sigma_{adm}} \rightarrow \frac{2{,}40.10^{-5}}{0{,}11162} = \frac{3{,}125\,P}{26.10^6} \rightarrow \boxed{P = 1{,}79\ kN/m}$$

. Verificação do cisalhamento.

$$Q = \frac{0{,}11162}{2} . 0{,}11162 . 0{,}03 \rightarrow Q = 1{,}87.10^{-4}\ m$$

$$\tau_{máx} = \frac{V_{máx}.\ Q}{I.\ t} = \frac{2{,}5 . 1788 . 1{,}87.10^{-4}}{2{,}40.10^{-5}.\ 0{,}03} \rightarrow \tau_{máx} = 1{,}16\,MPa < \tau_{adm} \rightarrow OK!$$

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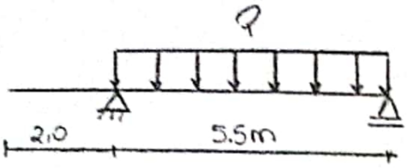
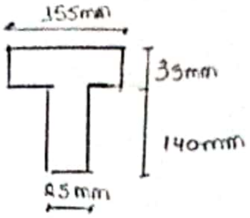
QUESTÃO 1 . TIPO D.





$\sigma_{adm} = 35\,MPa$

$\tau_{adm} = 1,80\,MPa$

- Reações de apoio.

$\circlearrowleft + \sum M_B = 0 \rightarrow P \cdot 5,50 \cdot 2,75 - V_A \cdot 5,50 = 0 \rightarrow$ $\boxed{V_A = V_B = 2,75\,P}$

- cálculo do centróide da seção.

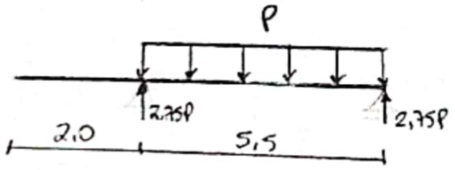
$$\bar{Y} = \frac{70 \cdot 140 \cdot 25 + 157,5 \cdot 155 \cdot 35}{140 \cdot 25 + 155 \cdot 35} \rightarrow \boxed{\bar{Y} = 123,19\,mm}$$

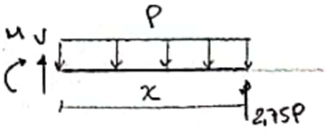
- Cálculo do centróide

$$I = \left[\frac{0,025 \cdot 0,14^3}{12} + (0,025 \cdot 0,14 \cdot 0,05319^2)\right] + \left[\frac{0,155 \cdot 0,035^3}{12} + (0,035 \cdot 0,155 \cdot 0,03431^2)\right]$$

$$\boxed{I = 2,26 \cdot 10^{-5}\,m^4}$$

- Momento fletor e esforço cortante máximo.





$V - P \cdot x + 2,75P = 0$

$V = Px - 2,75P$

$\boxed{V_{máx} = 2,75P}$

$$\sum M = 0 \rightarrow -M - \frac{Px^2}{2} + 2,75P = 0 \rightarrow M = 2,75P - \frac{Px^2}{2}$$

$$\boxed{M_{máx} = 3,781P}$$

- Cálculo do maior valor de P.

$$S_{req} = \frac{M_{máx}}{\sigma_{adm}} = \frac{I}{\bar{Y}} = \frac{M_{máx}}{\sigma_{adm}} \rightarrow \frac{2,26 \cdot 10^{-5}}{0,12319} = \frac{3,781\,P}{35 \cdot 10^6} \rightarrow \boxed{P = 1,70\,kN/m}$$

- Verificação do cisalhamento.

$$Q = \frac{0,12319}{2} \cdot 0,12319 \cdot 0,025 = 1,90 \cdot 10^{-4}\,m^3$$

$$\tau_{máx} = \frac{V_{máx} \cdot Q}{I \cdot t} = \frac{2,75 \cdot 1,70 \cdot 1,90 \cdot 10^{-4}}{2,26 \cdot 10^{-5} \cdot 0,025} \rightarrow \tau_{máx} = 1,57\,MPa < \tau_{adm} \rightarrow OK!$$

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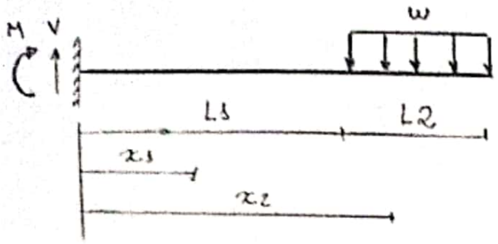
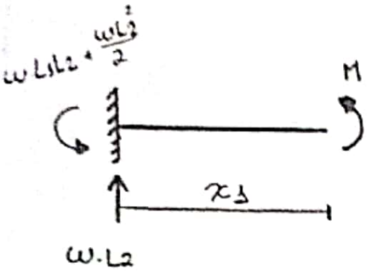
QUESTÃO 2.a



$\circlearrowleft \sum M_A = 0$

$$-M - w \cdot L_2 \cdot \left(L_1 + \frac{L_2}{2}\right) = 0 \rightarrow M = -w \cdot L_1 \cdot L_2 - \frac{w \cdot L_2^2}{2}$$

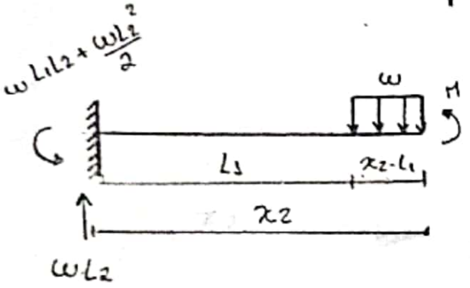
$$V = w \cdot L_2$$



$\circlearrowleft \sum M = 0$

$$M + w \cdot L_1 \cdot L_2 + \frac{w \cdot L_2^2}{2} - w \cdot L_2 \cdot x_1 = 0$$

$$\boxed{M_{(x_1)} = w \cdot L_2 \cdot x_1 - w \cdot L_1 \cdot L_2 - \frac{w \cdot L_2^2}{2}}$$



$\circlearrowleft \sum M = 0$

$$M + w(x_2 - L_1) \cdot \frac{(x_2 - L_1)}{2} - w \cdot L_2 x_2 + wL_1L_2 + \frac{wL_2^2}{2} = 0$$

$$M + \frac{wx_2^2}{2} - wx_2L_1 + \frac{wL_1^2}{2} - wL_2x_2 + wL_1L_2 + \frac{wL_2^2}{2} = 0$$

$$\boxed{M_{(x_2)} = wx_2L_1 - \frac{wx_2^2}{2} + wL_2x_2 - wL_1L_2 - \frac{w \cdot L_2^2}{2} - \frac{wL_1^2}{2}}$$

Integrais

$$\theta_{(x_1)} = \frac{w \cdot L_2 x_1^2}{2} - w L_1 L_2 x_1 - \frac{w \cdot L_2^2 x_1}{2} + C_1$$

$$\upsilon_{(x_1)} = \frac{w \cdot L_2 x_1^3}{6} - \frac{w \cdot L_1 L_2 x_1^2}{2} - \frac{w \cdot L_2^2 x_1^2}{4} + C_1 x_1 + C_2$$

$$\theta_{(x_2)} = \frac{w \cdot L_1 x_2^2}{2} - \frac{w x_2^3}{6} + \frac{w L_2 x_2^2}{2} - w \cdot L_1 L_2 x_2 - \frac{w \cdot L_2^2 x_2}{2} - \frac{w \cdot L_1^2 x_2}{2} + C_3$$

$$\upsilon_{(x_2)} = \frac{w \cdot L_1 x_2^3}{6} - \frac{w \cdot x_2^4}{24} + \frac{w \cdot L_2 x_2^3}{6} - \frac{w \cdot L_1 L_2 x_2^2}{2} - \frac{w \cdot L_2^2 \cdot x_2^2}{4} - \frac{w \cdot L_1^2 \cdot x_2^2}{4} + C_3 x_2 + C_4$$

condições de contorno e continuidade:

I) Para $x_1 = 0$, temos $\upsilon_{(x_1)} = 0$ e $\theta_{(x_1)} = 0$

II) Para $x_1 = x_2 = L_1$, temos $\theta_{(x_1)} = \theta_{(x_2)}$

III) Para $x_1 = x_2 = L_1$, temos $\upsilon_{(x_1)} = \upsilon_{(x_2)}$

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Analisando a condição I

$C_1 = 0$ e $C_2 = 0$

Analisando a condição II

$$\frac{w L_2 L_1^2}{2} - w L_1^2 L_2 - \frac{w L_2^2 L_1}{2} = \frac{w L_1^3}{2} - \frac{w L_1^3}{6} + \frac{w L_2 L_1^2}{2} - w L_1^2 L_2 - \frac{w L_2^2 L_1}{2} - \frac{w L_1^3}{2} + C_3$$

$$C_3 + \frac{w L_1^3}{2} - \frac{w L_1^3}{6} - \frac{w L_1^3}{2} = 0 \rightarrow \boxed{C_3 = \frac{w L_1^3}{6}}$$

Analisando a condição III:

$$\frac{w L_2 L_1^3}{6} - \frac{w L_1^3 L_2}{2} - \frac{w L_1^2 L_2^2}{4} = \frac{w L_1^4}{6} - \frac{w L_1^4}{24} + \frac{w L_2 L_1^3}{6} - \frac{w L_1^3 L_2}{2} - \frac{w L_2^2 L_1^2}{4} - \frac{w L_1^4}{4} + \frac{w L_1^4}{6} + C_4$$

$$C_4 + \frac{w L_1^4}{6} - \frac{w L_1^4}{24} - \frac{w L_1^4}{4} + \frac{w L_1^4}{6} = 0 \rightarrow \boxed{C_4 = -\frac{w L_1^4}{24}}$$

Equações finais:

$$\theta_{(x_1)} = \frac{w L_2 x_1^2}{2} - w L_1 L_2 x_1 - \frac{w L_2^2 x_1}{2} \rightarrow \boxed{\theta_{(x_1)} = \frac{w L_2 x_1}{2EI}\left(x_1 - 2L_1 - L_2\right)}$$

$$\vartheta_{(x_1)} = \frac{w L_2 x_1^3}{6} - \frac{w L_1 L_2 x_1^2}{2} - \frac{w L_2^2 x_1^2}{4} \rightarrow \boxed{\vartheta_{(x_1)} = \frac{w L_2 x_1^2}{6EI}\left(x_1 - 3L_1 - \frac{3}{2} L_2\right)}$$

$$\theta_{(x_2)} = \frac{w L_1 x_2^2}{2} - \frac{w x_2^3}{6} + \frac{w L_2 x_2^2}{2} - w L_1 L_2 x_2 - \frac{w L_2^2 x_2}{2} - \frac{w L_1^2 x_2}{2} + \frac{w L_1^3}{6}$$

$$\boxed{\theta_{(x_2)} = \frac{w}{6EI}\left(3L_1 x_2^2 - x_2^3 + 3L_2 x_2^2 - 6L_1 L_2 x_2 - 3L_2^2 x_2 - 3L_1^2 x_2 + L_1^3\right)}$$

$$\vartheta_{(x_2)} = \frac{w L_1 x_2^3}{6} - \frac{w x_2^4}{24} + \frac{w L_2 x_2^3}{6} - \frac{w L_1 L_2 x_2^2}{2} - \frac{w L_2^2 x_2^2}{4} - \frac{w L_1^2 x_2^2}{4} + \frac{w L_1^3 x_2}{6} - \frac{w L_1^4}{24}$$

$$\boxed{\vartheta_{(x_2)} = \frac{w}{24EI}\left(4L_1 x_2^3 - x_2^4 + 4L_2 x_2^3 - 12 L_1 L_2 x_2^2 - 6L_2^2 x_2^2 - 6L_1^2 x_2^2 + 4L_1^3 x_2 - L_1^4\right)}$$

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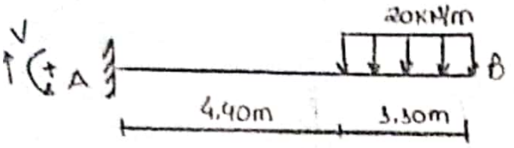
QUESTÃO 2B - TIPO A



$\sigma_{adm} = 190MPa$

$\tau_{adm} = 10MPa$

$\circlearrowleft + \sum M_A = 0$

$-20 \cdot 1,10 \cdot 4,95 + M = 0$

$\boxed{M = 108,90 kN \cdot m}$

$V - 20 \cdot 1,10 = 0 \rightarrow \boxed{V = 22 kN}$

$S_{req} = \frac{M_{máx}}{\sigma_{adm}} = \frac{108,90 \cdot 10^3}{190 \cdot 10^6} \rightarrow \boxed{S_{req} = 5,73 \cdot 10^{-4} m^3 \underset{\times 10^9}{=} 573 \cdot 10^3 mm^3}$

Possíveis Perfis:

W 410 x 39 → $S = 632 \cdot 10^3 mm^3$

W 360 x 39 → $S = 578 \cdot 10^3 mm^3$

W 310 x 67 → $S = 948 \cdot 10^3 mm^3$

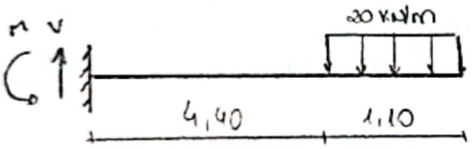
W 250 x 58 → $S = 693 \cdot 10^3 mm^3$

W 200 x 59 → $S = 583 \cdot 10^3 mm^3$

→ Perfil correto → W 360 x 39

$\boxed{\tau_{máx} = \frac{22 \cdot 10^3}{353 \cdot 6,48} \Rightarrow \tau_{máx} = 9,62 MPa < \tau_{adm} \text{ OK}}$

QUESTÃO 2B - TIPO B.



$\sigma_{adm} = 160MPa.$

$\tau_{adm} = 10MPa$

$\circlearrowleft + \sum M_A = 0$

$-20 \cdot 1,10 \cdot 4,95 + M = 0$

$\boxed{M = 108,90 kN \cdot m}$

$V - 20 \cdot 1,10 = 0 \rightarrow \boxed{V = 22 kN}$

$S_{req} = \frac{M_{máx}}{\sigma_{adm}} = \frac{108,90 \cdot 10^3}{160 \cdot 10^6} \rightarrow \boxed{S_{req} = 6,81 \cdot 10^{-4} m^3 = 681 \cdot 10^3 mm^3}$

Possíveis perfis:

W 410 x 46 → $S = 774 \cdot 10^3 mm^3$

W 360 x 45 → $S = 688 \cdot 10^3 mm^3$

W 310 x 67 → $S = 948 \cdot 10^3 mm^3$

W 250 x 58 → $S = 693 \cdot 10^3 mm^3$

W 200 x 71 → $S = 709 \cdot 10^3 mm^3$

→ Perfil correto → W 360 x 45

$\boxed{\tau_{máx} = \frac{22 \cdot 10^3}{352 \cdot 6,86} \rightarrow \tau_{máx} = 9,11 MPa < \tau_{adm} \text{ ok!}}$

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